Leveraging Linear Programming for pseudo-Boolean solving

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Three abbreviations

- CP = constraint programming
- PB = pseudo-Boolean
- LP = linear programming

CP demo!

- Pigeonhole problem in IDP
  - http://dtai.cs.kuleuven.be/krr/idp-ide/?src=c01635bf2172be67577f0856684fb3f8
  - Timeout on small problem sizes

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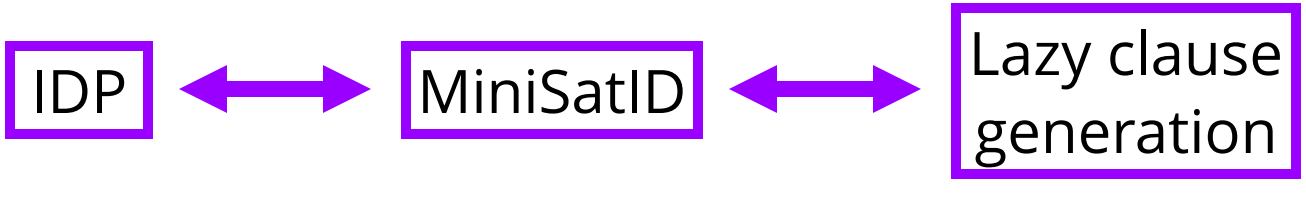
- Why is IDP's performance this bad?

Explanation: *resolution* is bad at pigeonhole

- IDP uses MiniSatID as backend CP solver
- MiniSatID uses *lazy clause generation* algorithm



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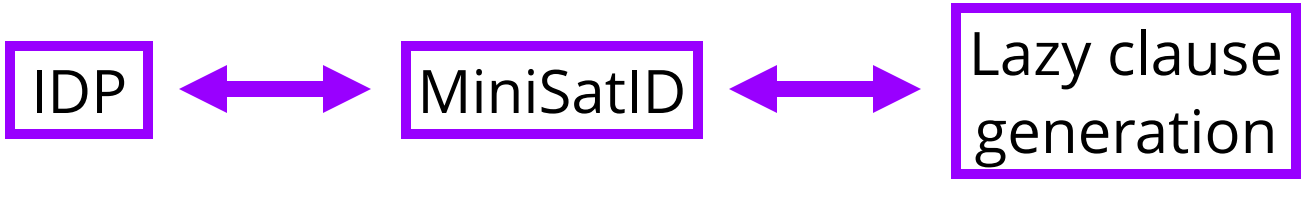

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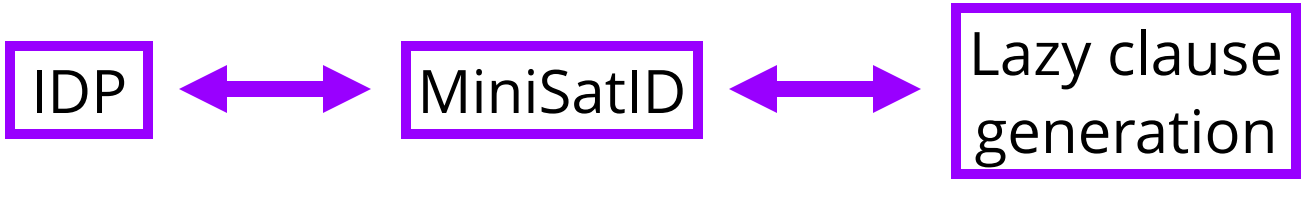

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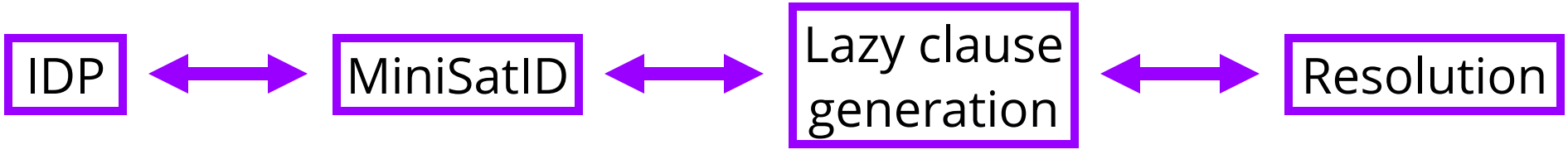

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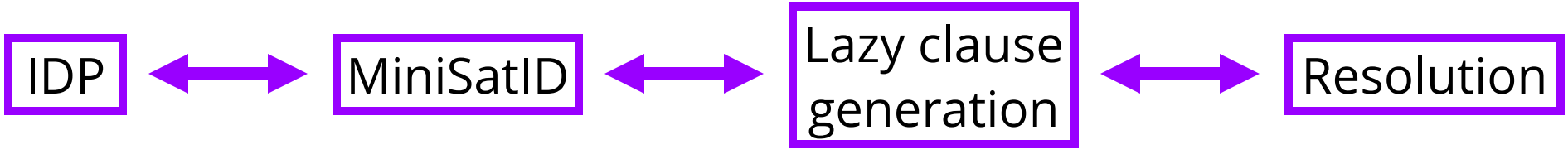

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- RoundingSat has no problem with pigeonhole 
- RoundingSat fails on several other rationally infeasible problems 

Summary so far

1. CP and PB solvers struggle on rational infeasibility
2. Stronger underlying proof system helps on some, but not all problems
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How to exploit rational infeasibility during search?

Modern search loop

Propagation

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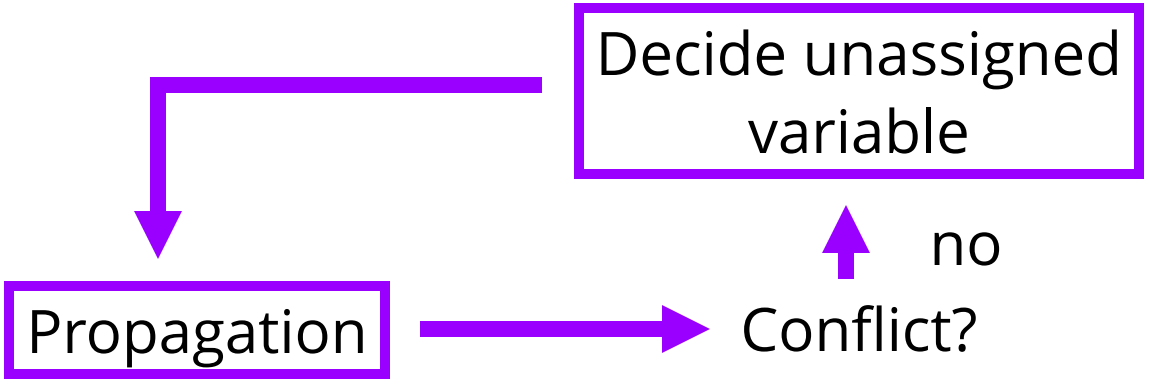

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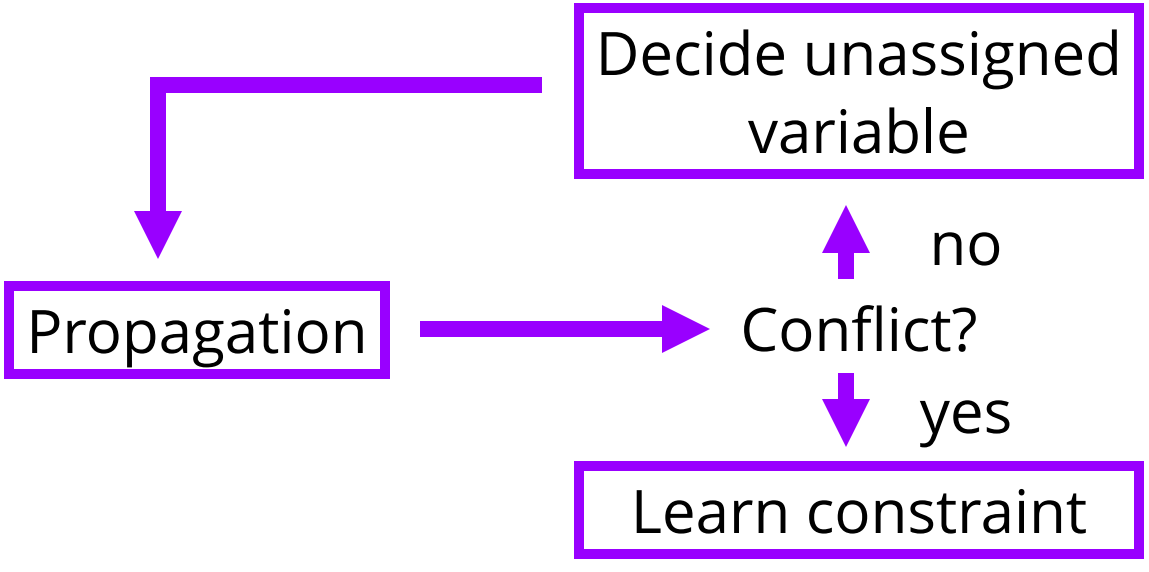

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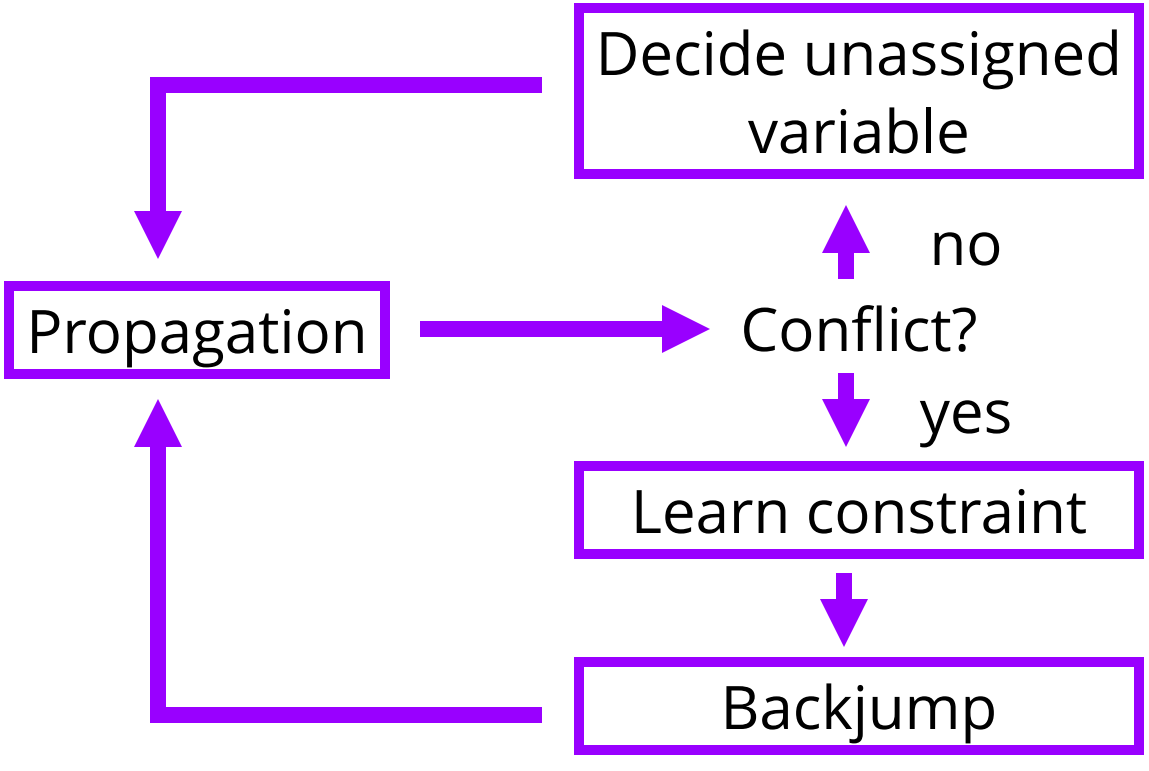

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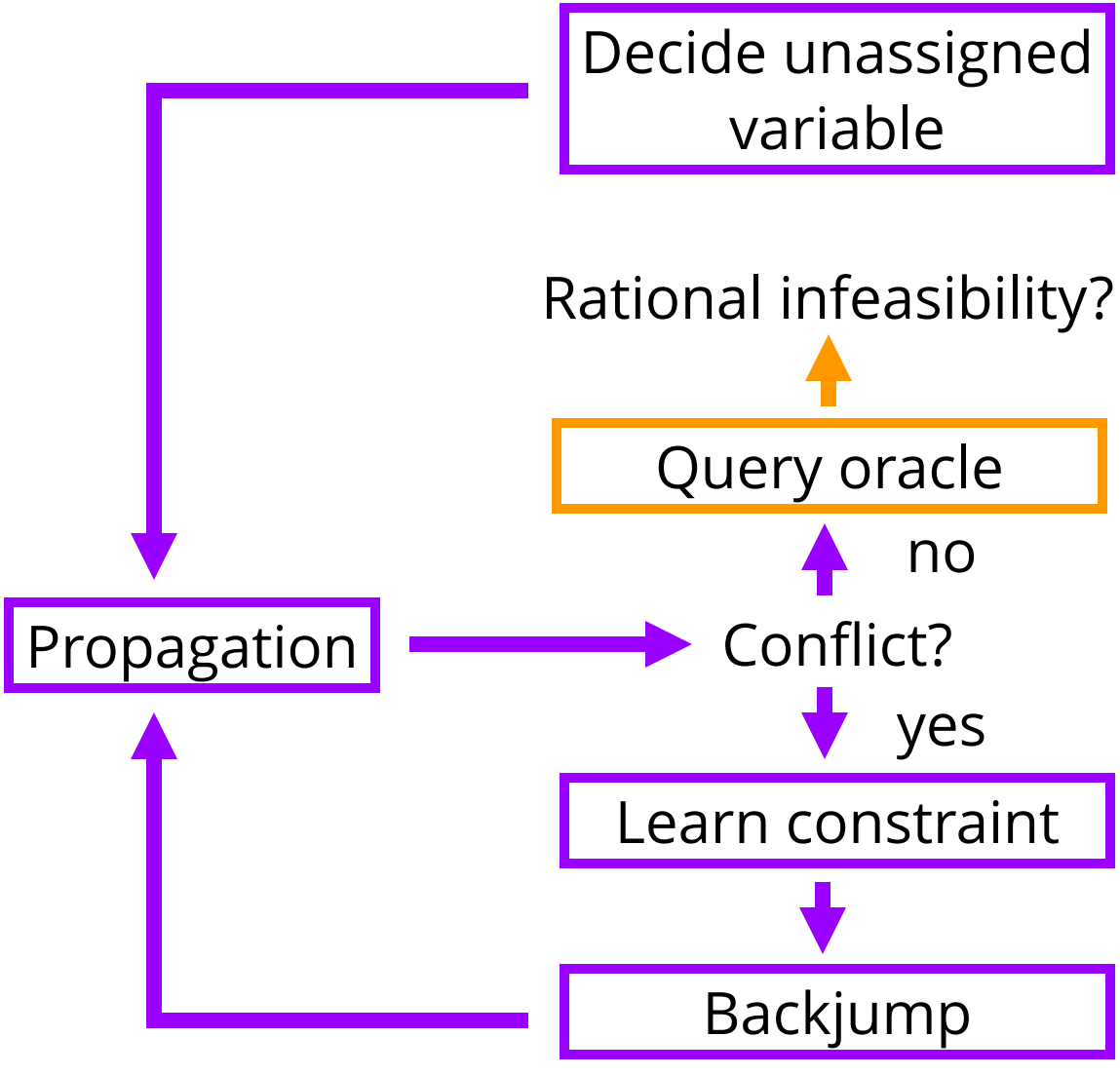

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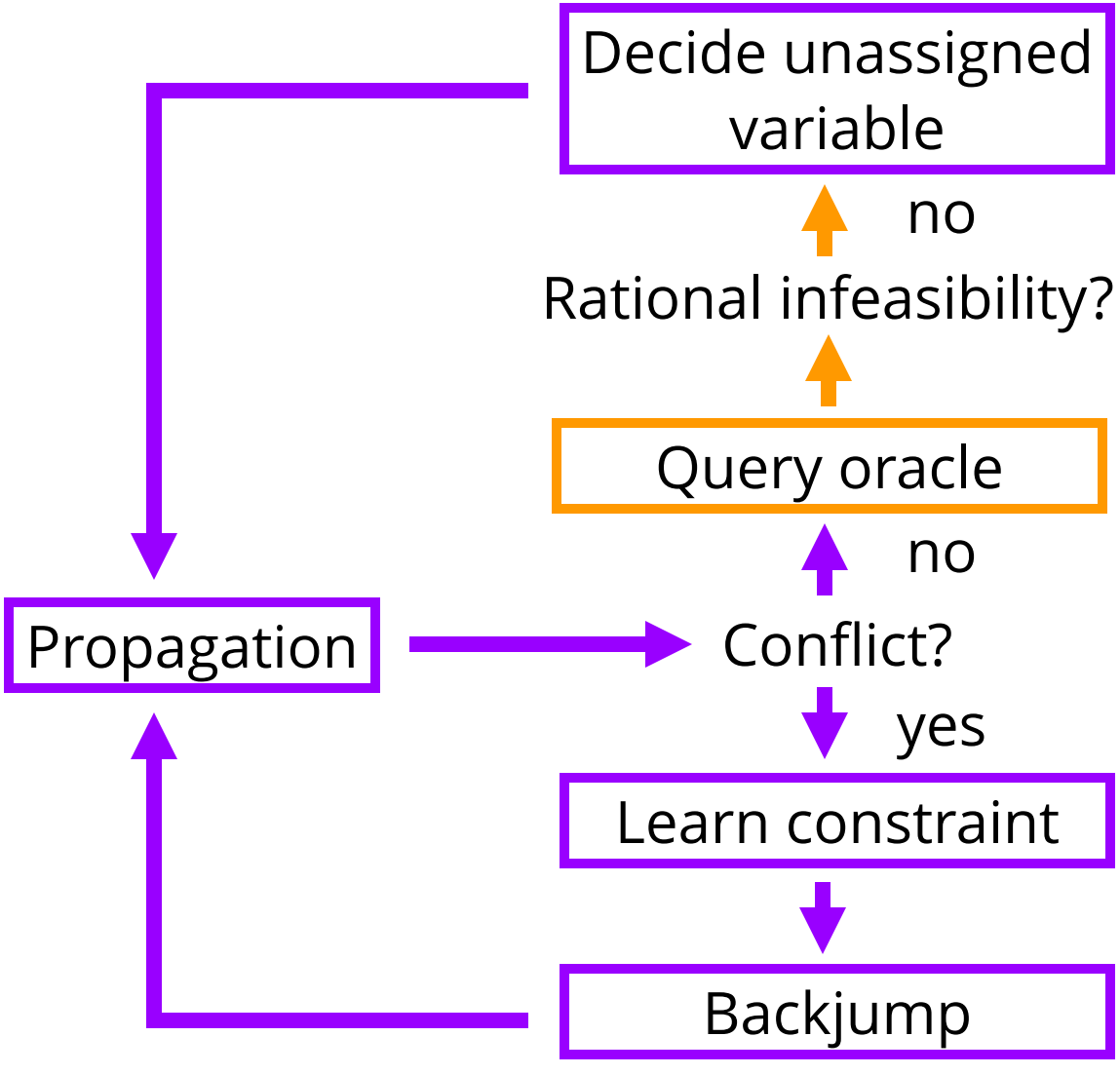

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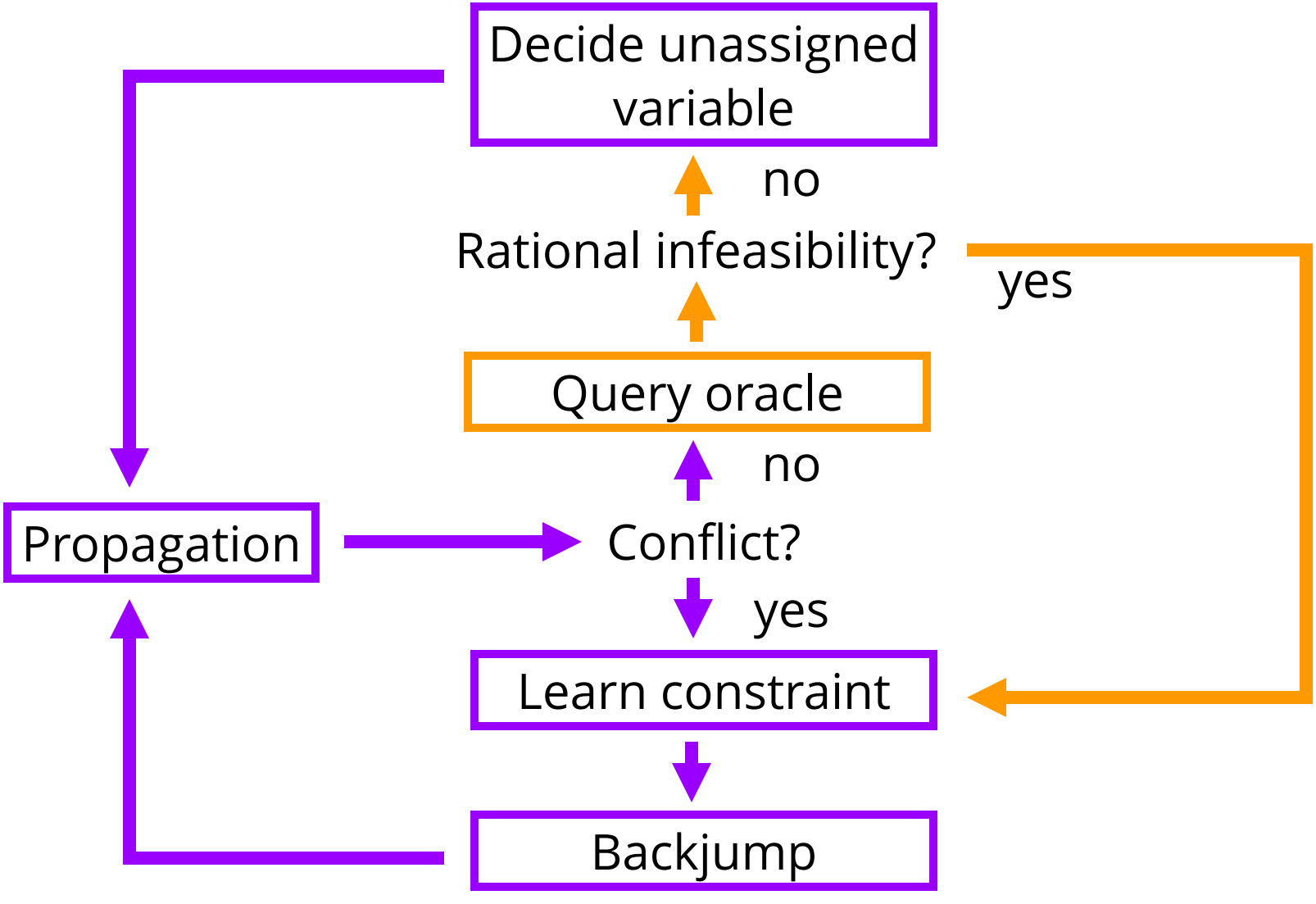

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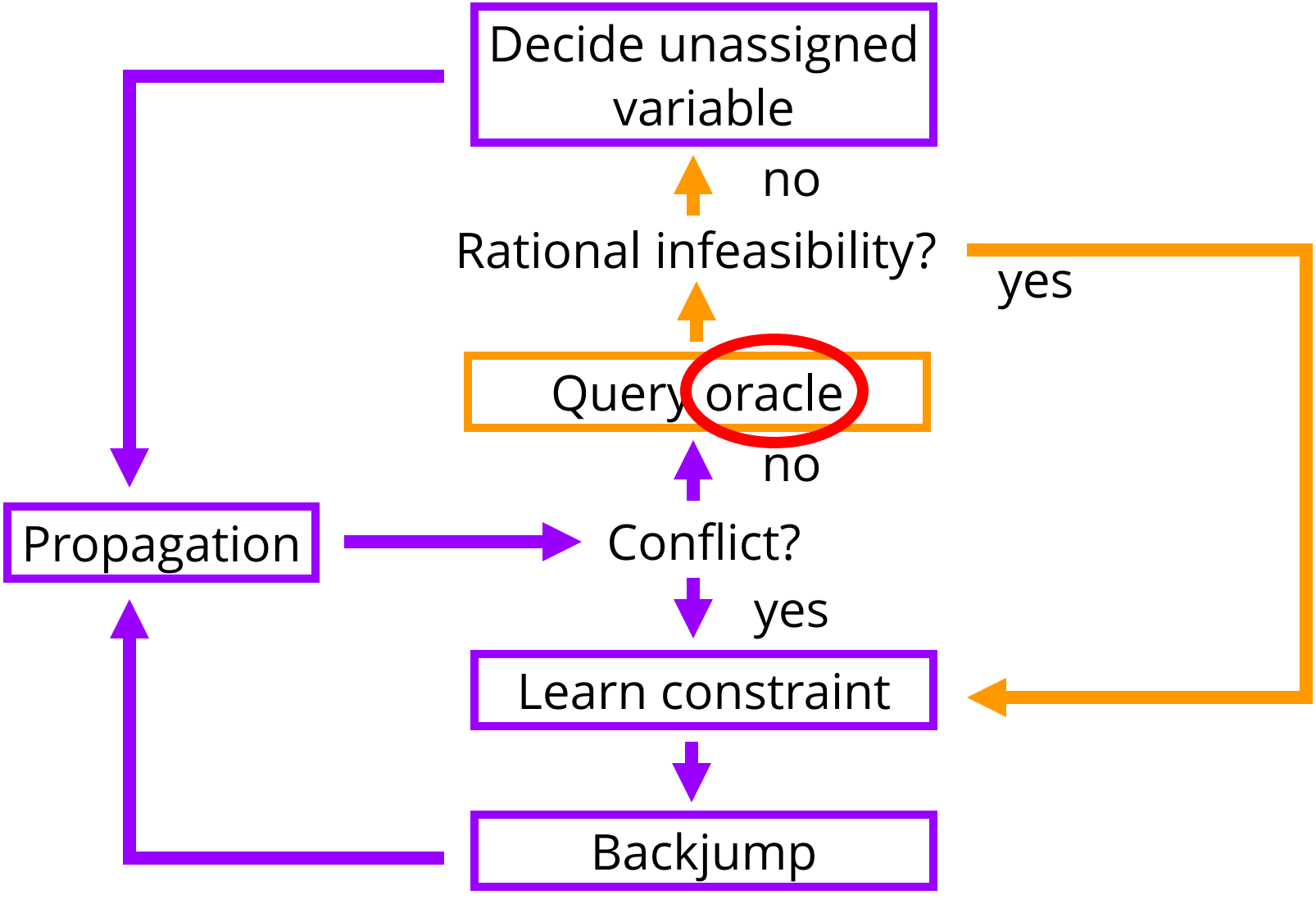

Linear Programming (LP) solvers

- In:
  - conjunction of linear constraints
  - variable bounds
  - objective function

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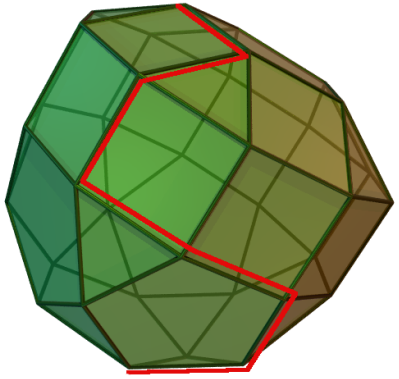

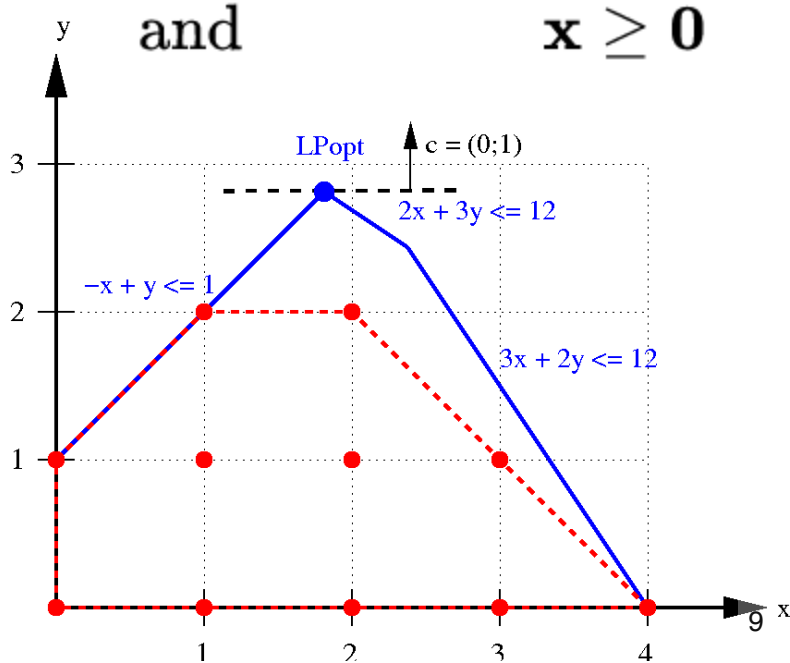

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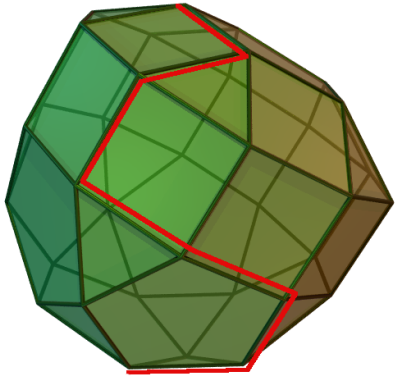

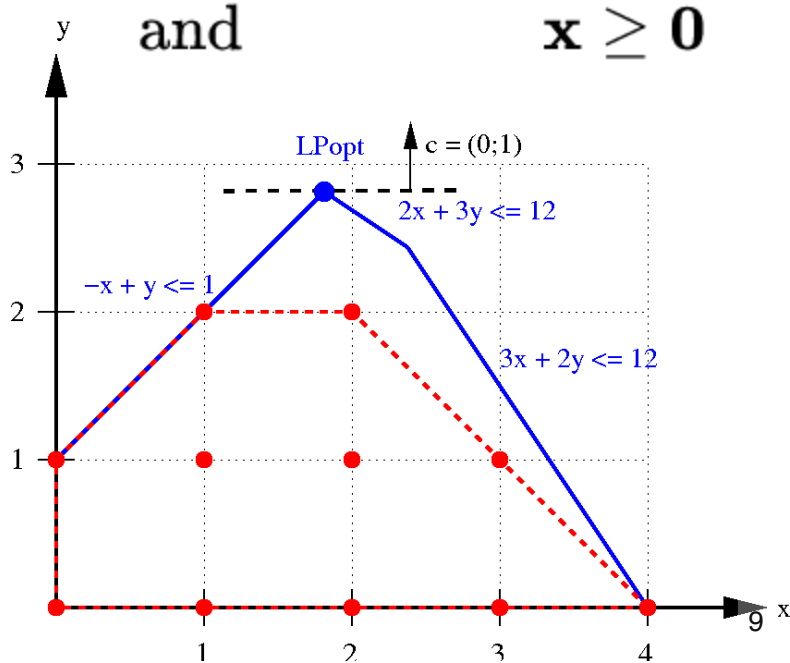

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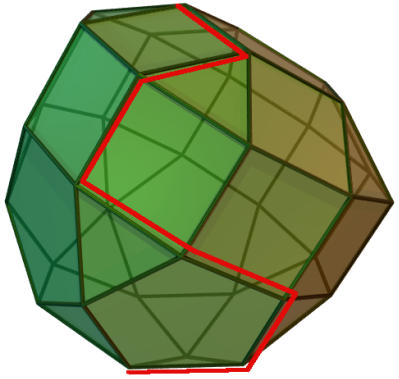

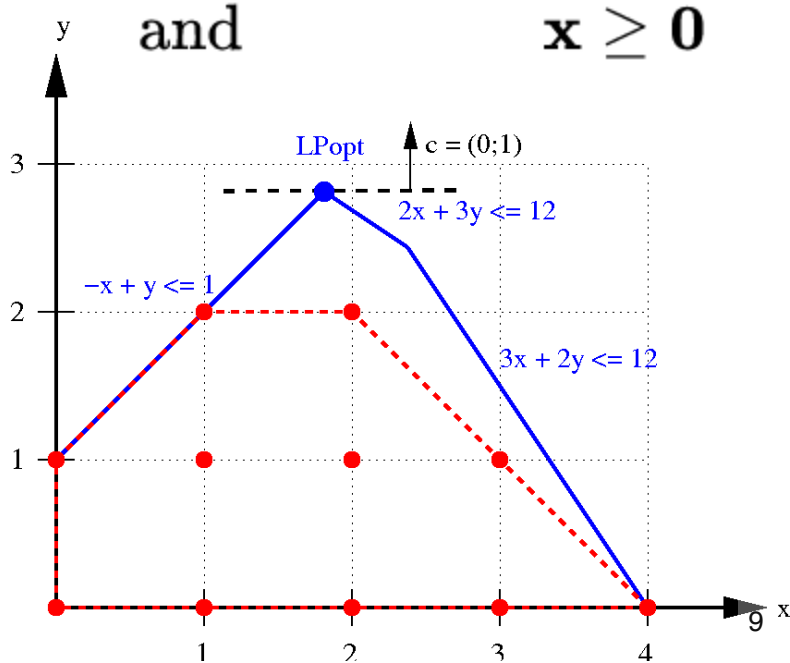

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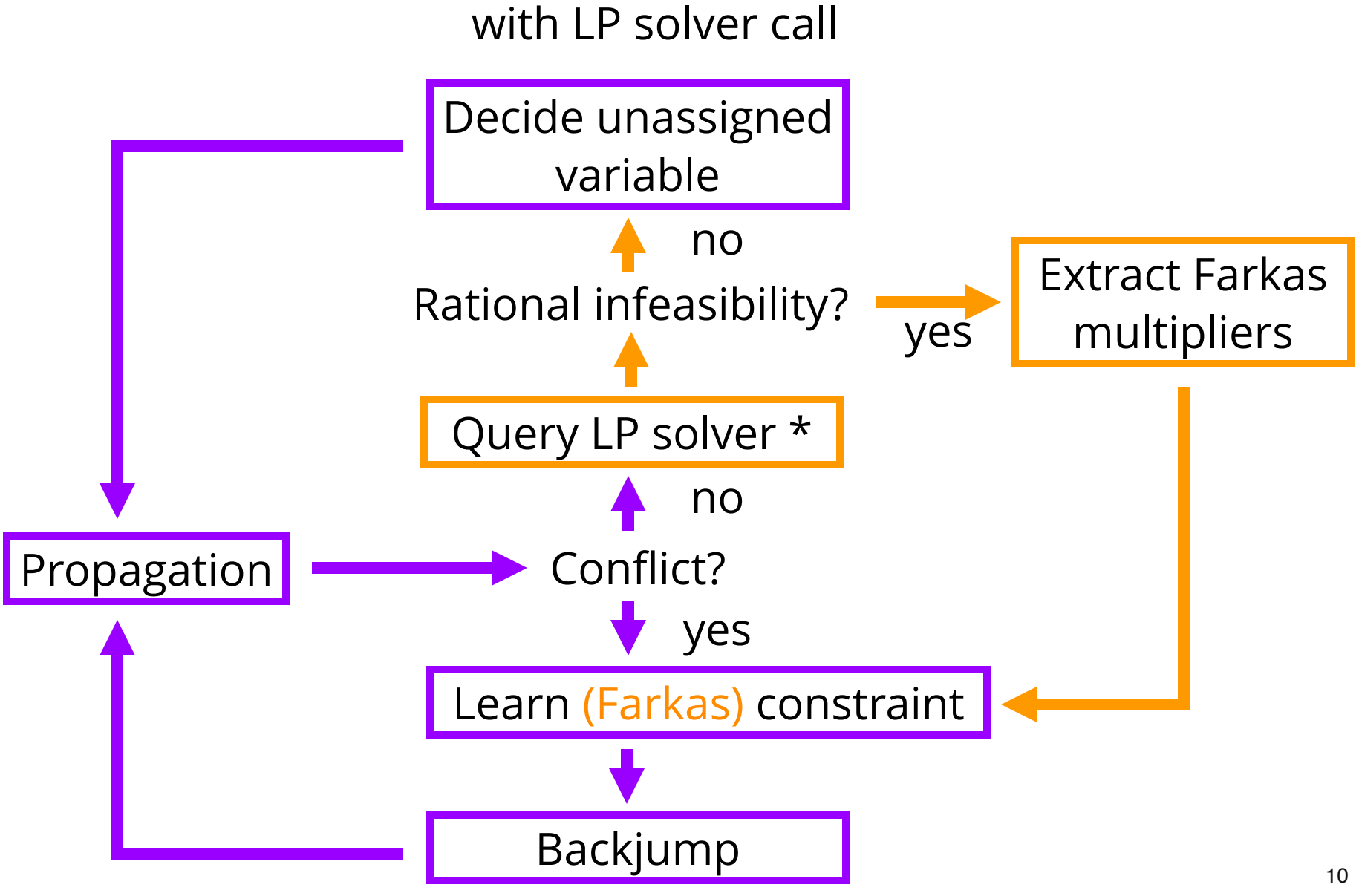

Two technical problems

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- LP solvers are relatively slow
  - Limit calls to LP solver
  - Limit LP solver running time
- LP solvers use inexact floating point arithmetic
  - Independently calculate Farkas constraint with exact multiple precision
  - Verify falsifiedness of Farkas constraint

Working implementation with PB solver

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Experiments!

- 5 solver configurations
  - RoundingSat
  - **RoundingSat+SoPlex**
  - SCIP
  - Sat4J
  - Sat4J-CP
- 3000s on 16GiB machines
- 4 benchmark families:
  - PB12
  - PB16
  - MIPLIB
  - PROOF

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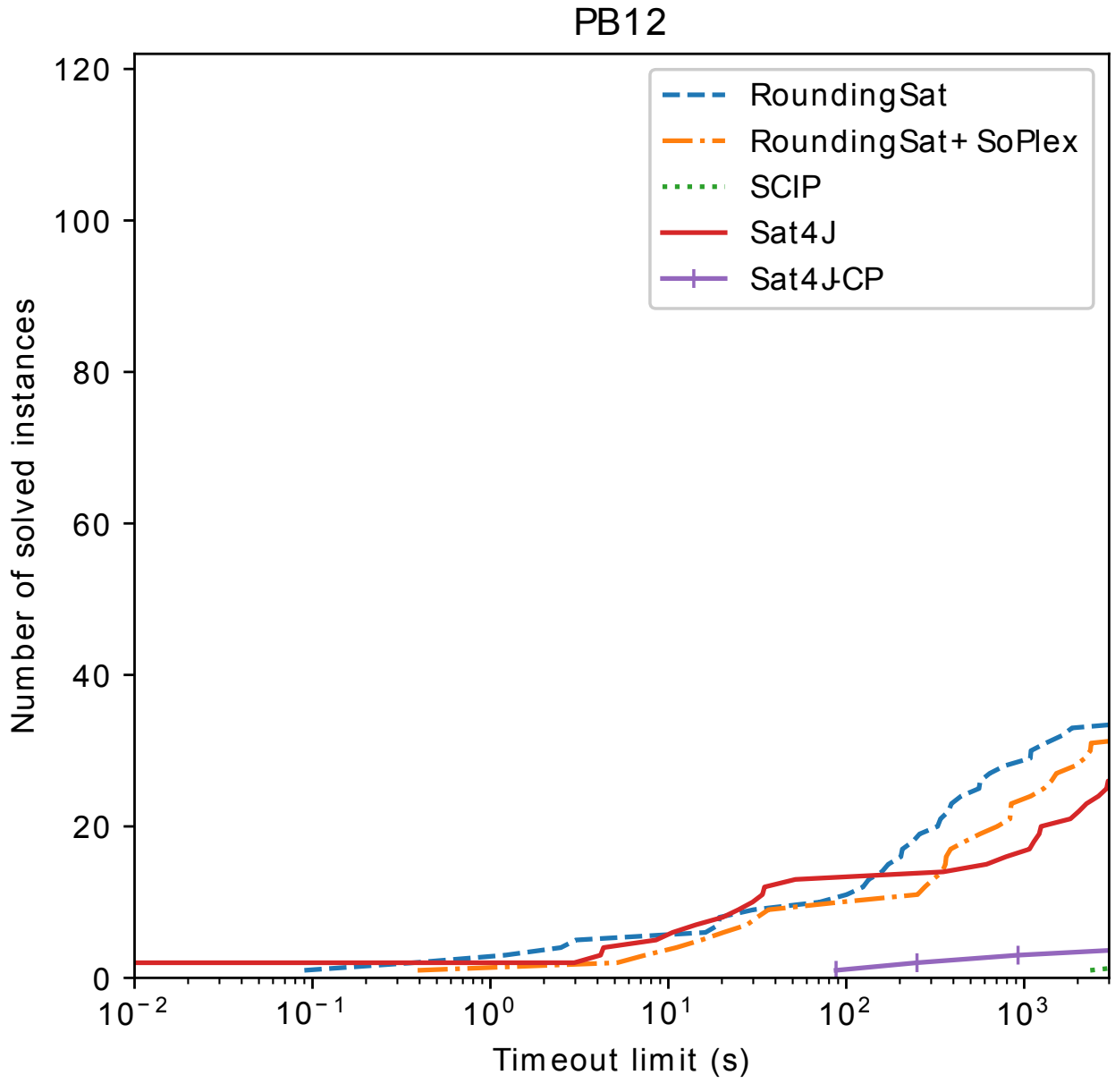

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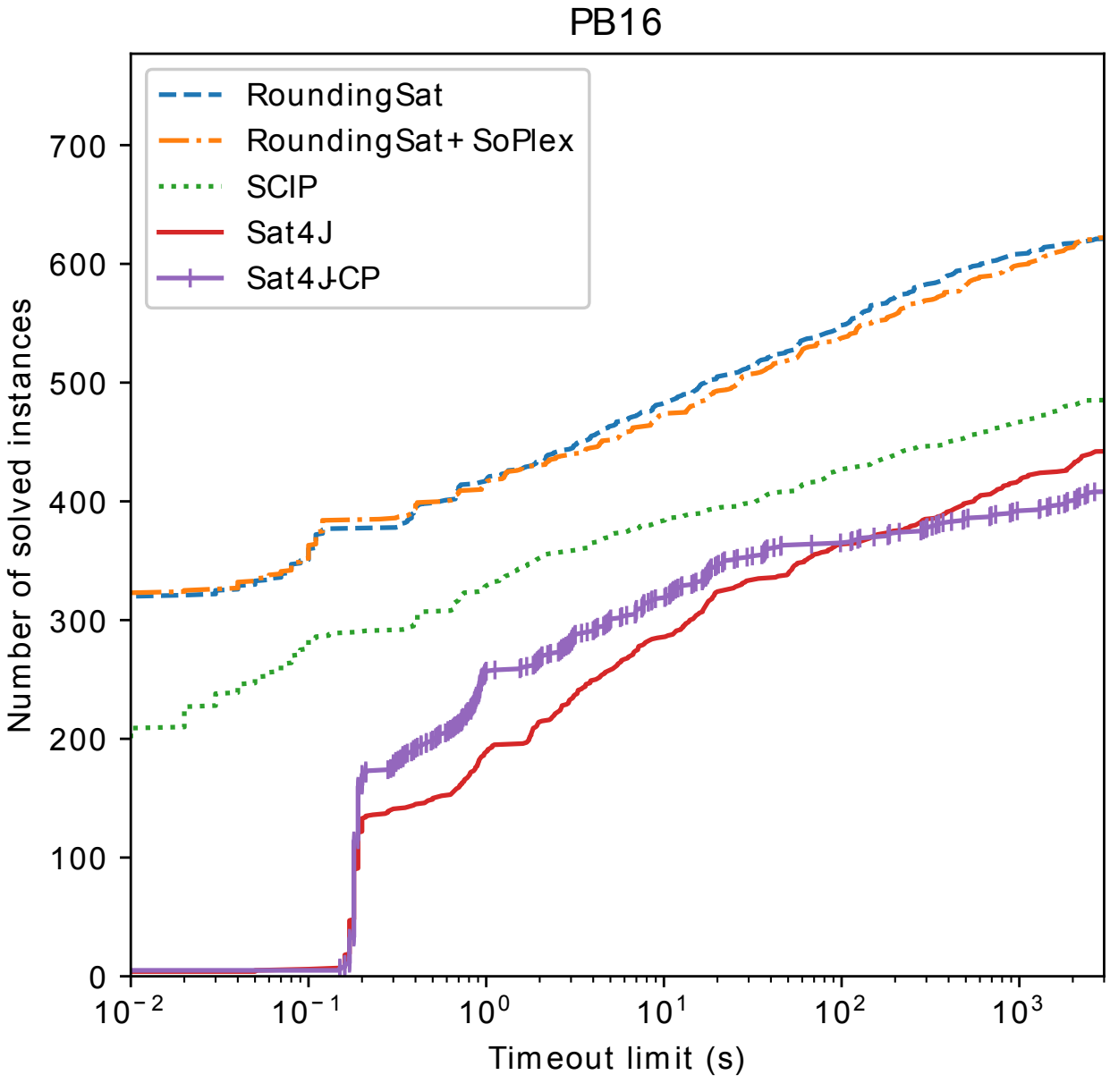

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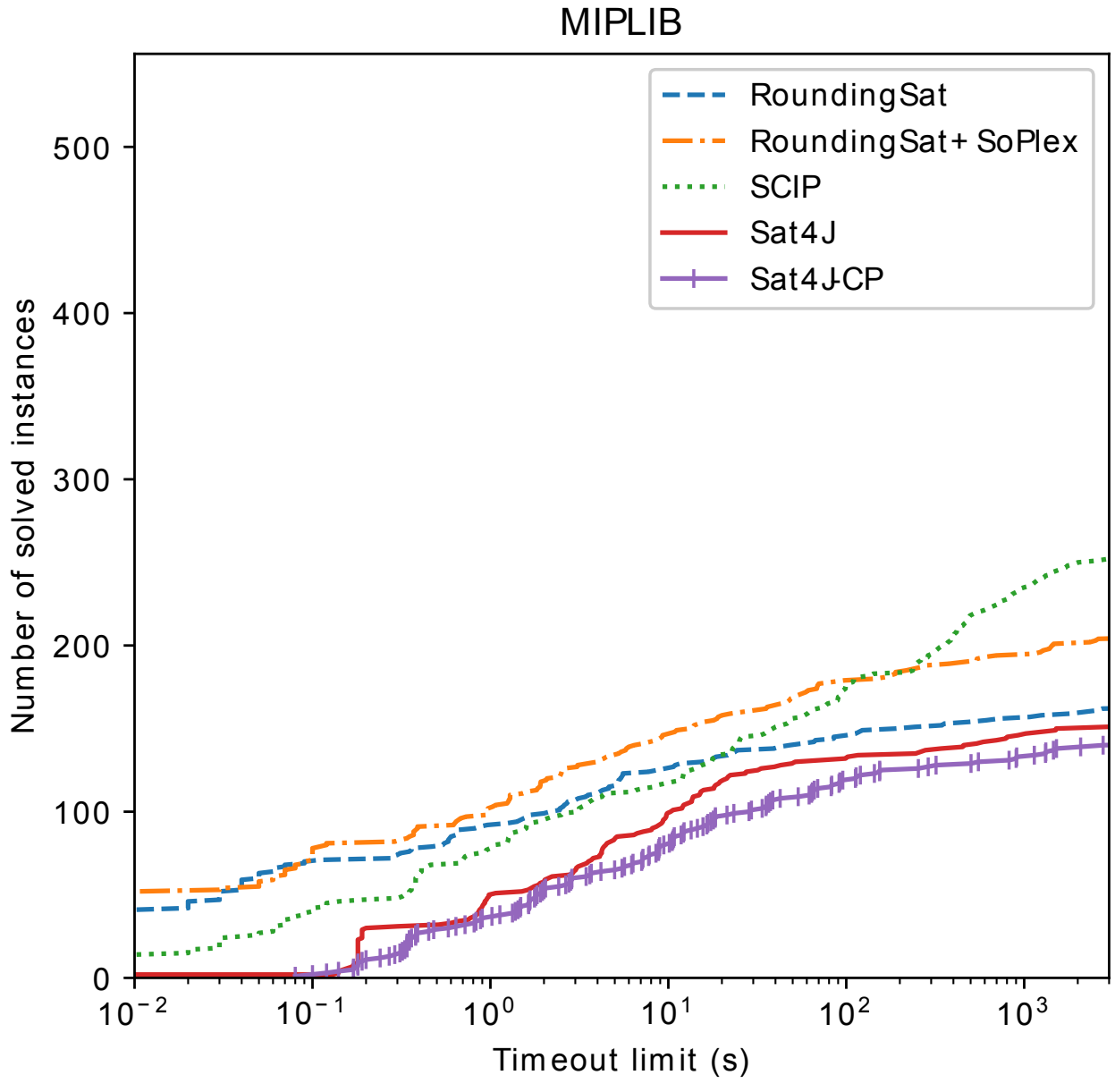

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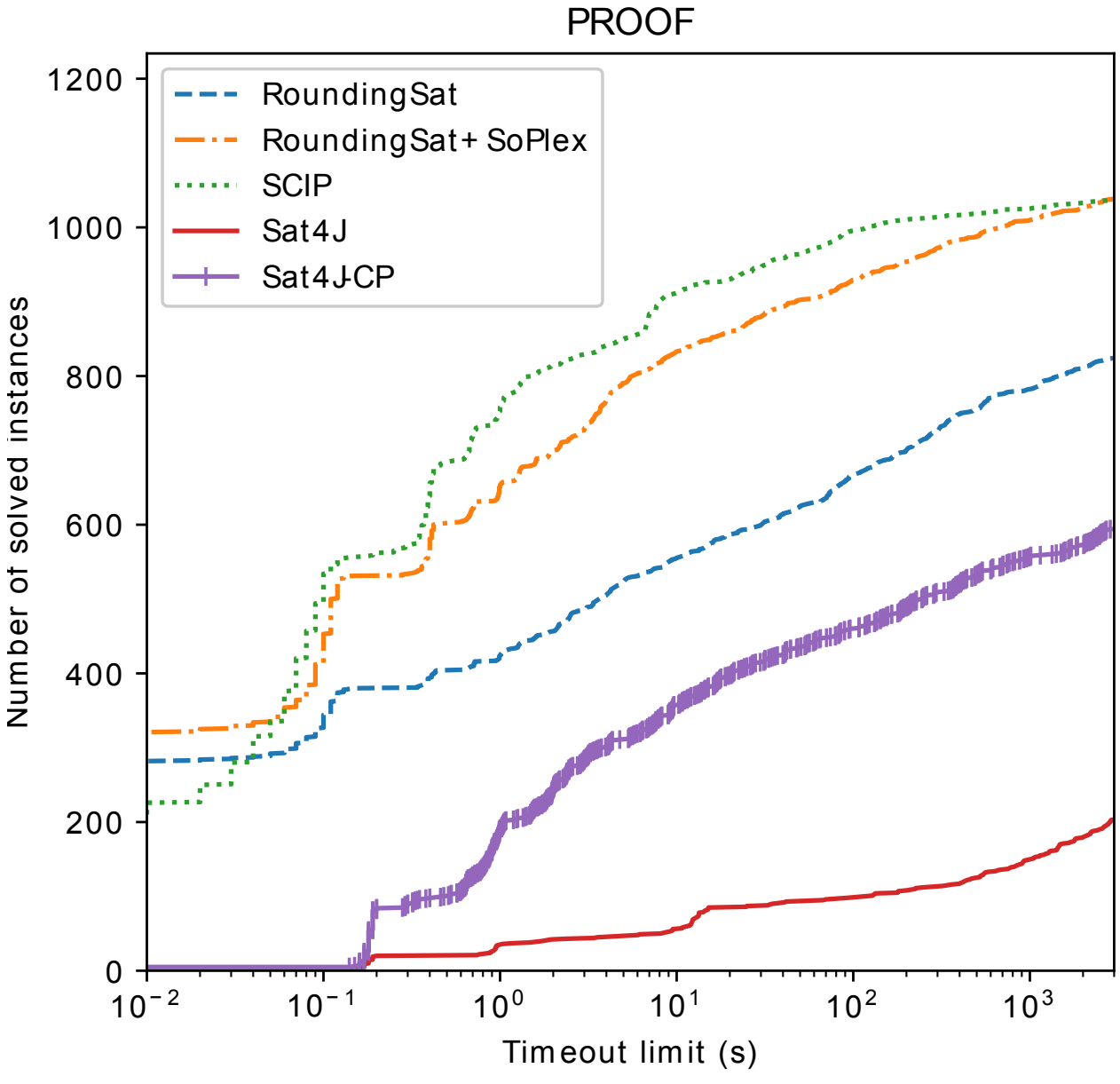

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- SoPlex does not like PB12

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- Optimization!

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Questions to NordConsNet:

- would LP integration be helpful for CP solvers?
- does any CP solver do this already?

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Thanks for your attention!
Questions?

References

[1] The intractability of resolution - 1985 - Haken

[2] Über die Theorie der Einfachen Ungleichungen - 1902 - Farkas

[3] A polynomial algorithm for linear programming - 1979 - Khachiyan

[4] The Many Roads Leading to Rome: Solving Zinc Models by Various Solvers - 2008 - Becket e.a.